ECCOMAS (ECCM-ECFD) 2018 conference, Glasgow, 11-15 June 2018

Wall-resolved and wall-modeled ILES based on high-order DG

Ralf Hartmann

14 June 2018

DLR

Overview

1. DG discretization of
   - flow equations
   - integral quantities and
   - local quantities
2. Discretization settings and results/examples for
   - RANS
   - ILES
3. Wall-modelled ILES (WM-ILES)
   - Approach
   - Results

Discretization of equations, integral and local quantities

Discretization of flow equations:

$$\int_\Omega \left(-\mathcal{F}^c(\mathbf{u}_h) + \mathcal{F}^v(\mathbf{u}_h, \nabla_h \mathbf{u}_h)\right) \,:\, \nabla_h \mathbf{v}_h \,\mathrm{d}\mathbf{x} + \ldots + \int_\Gamma \left(\hat{\mathbf{h}}_{\Gamma,h} - \underline{\hat{\sigma}}_{\Gamma,h}\mathbf{n}\right) \cdot \mathbf{v}_h \,\mathrm{d}s$$

Adjoint consistent discretization of **integral** quantities (drag, lift coefficients):

$$J(\mathbf{u}) = \int_{\Gamma_w} (p\,\mathbf{n} - \underline{\tau}\,\mathbf{n}) \cdot \psi \,\mathrm{d}s, \qquad J_h(\mathbf{u}_h) = \int_{\Gamma_w} \left(\hat{\mathbf{h}}_{\Gamma,h} - \underline{\hat{\sigma}}_{\Gamma,h}\mathbf{n}\right) \cdot \tilde{\psi} \,\mathrm{d}s.$$

Adjoint consistent discretization of **local** quantities (surface pressure, skin friction)

$$c_\mathrm{p}(\mathbf{u}) = \frac{p(\mathbf{u}) - p_\infty}{\frac{1}{2}\rho_\infty v_\infty^2}, \qquad c_{\mathrm{p},h}(\mathbf{u}_h) = \frac{\hat{\mathbf{h}}_{\Gamma,h} \cdot \tilde{\mathbf{n}} - p_\infty}{\frac{1}{2}\rho_\infty v_\infty^2},$$

$$c_\mathrm{f}(\mathbf{u}, \nabla\mathbf{u}) = \frac{\tau_W(\mathbf{u}, \nabla\mathbf{u})}{\frac{1}{2}\rho_\infty v_\infty^2}, \qquad c_{\mathrm{f},h}(\mathbf{u}_h, \nabla\mathbf{u}_h) = -\frac{\left(\underline{\hat{\sigma}}_{\Gamma,h}\mathbf{n}\right) \cdot \tilde{\mathbf{t}}}{\frac{1}{2}\rho_\infty v_\infty^2},$$

with $\tilde{\mathbf{n}} = (0, n_1, n_2, 0)^\top$ for the normal vector $\mathbf{n} = (n_1, n_2)^\top$,
and $\tilde{\mathbf{t}} = (0, t_1, t_2, 0)^\top$ for the tangential vector $\mathbf{t} = (t_1, t_2)^\top$.

Hartmann, Leicht: JCP 300, 754-778, 2015

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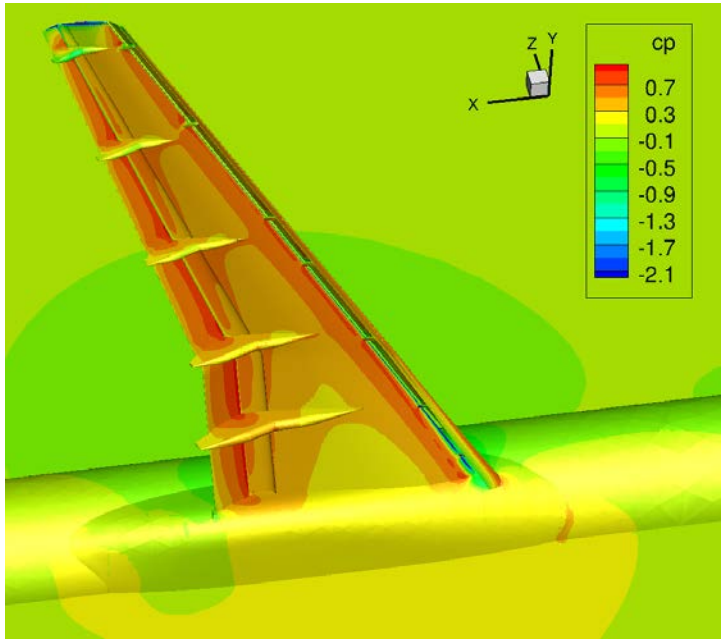
Computational/meshing challenge
4th Int. Workshop on High-Order CFD Meth.



DLR-F11 high lift configuration

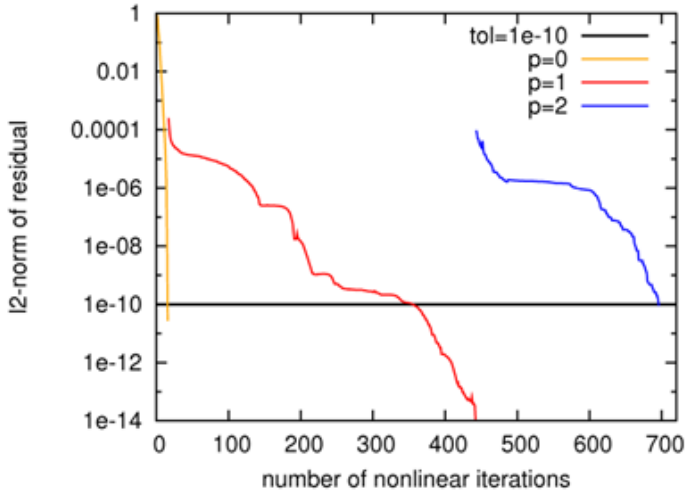
- Config 4 incl. slat tracks and flap track fairings
- Mach=0.175, Re=15.1x10^6, alpha=7.0°
- Fully turbulent (**RANS with Wilcox-kω model**)

Grid (by Harlan McMorris, CentaurSoft):

- Quadratic curved grid (Centaur)
- Hybrid grid (with prisms, pyramids and tetrahedra)
- 3.52x10^6 elements

Flow solver:

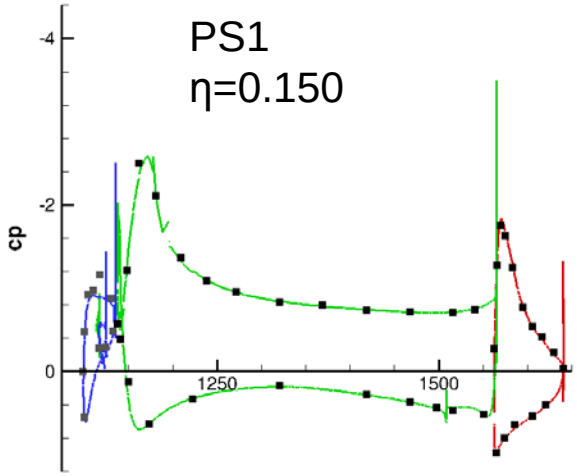
- PADGE solver, fully implicit solver (no multigrid)
- p=0, 1, 2 (3.5, 14.1, 35.2x10^6 DoFs/eqn)
- Convergence of nonlinear residual below 1e-10

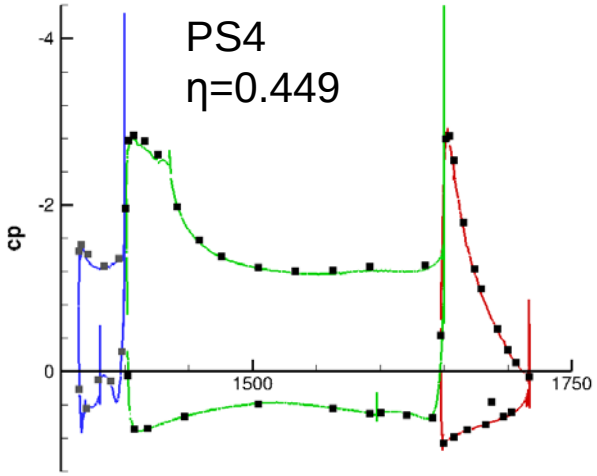


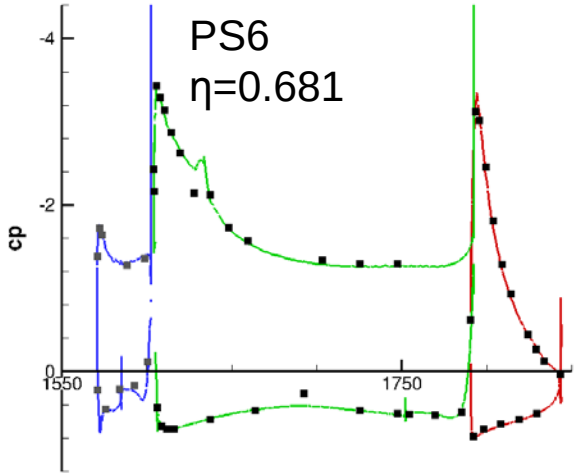
Hartmann, McMorris, Leicht: ECCOMAS 2016

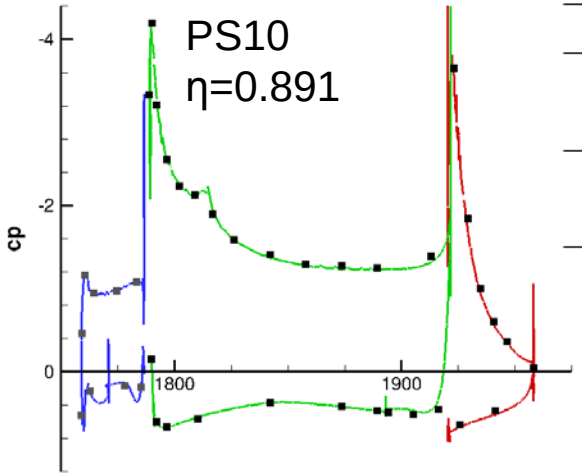
DLR

The DLR-F11 high lift configuration: Results









DLR-F11, Config 4
Mach=0.175
Re=15.1x10^6
alpha=7.0°
RANS Wilcox-kw
p=2: 35.2x10^6 DoFs/eqn

	C_L	C_D	C_M
Exp.	1.9270	0.1615	-0.5390
TAU*	1.8794	0.1681	-0.5647
	-2.5%	+4.1%	-4.8%
PADGE	1.8781	0.1649	-0.5704
	-2.5%	+2.1%	-5.8%

* Rudnik, Melber(SAO)
HiLiftPW-2, 2013

Hartmann, McMorris, Leicht: ECCOMAS 2016

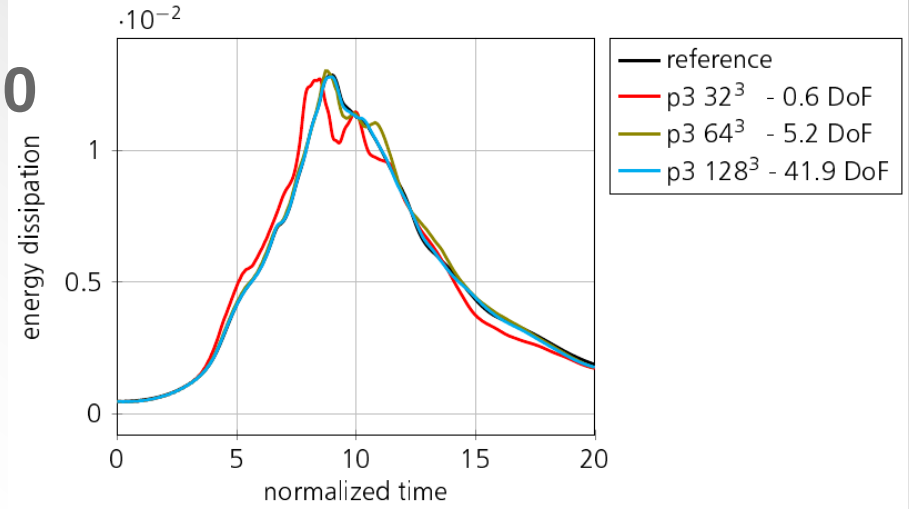
Discretization details: RANS vs. ILES

	RANS	ILES
Convective numerial flux	Roe with entropy fix	Roe without entropy fix
BR2 constant (on hexes)	6	2
DG basis functions [difference due to different flow solvers]	parametric	non-parametric

Taylor Green Vortex at Re=1600

DG, p=3 on 64^3 mesh,
Comparison explicit vs. implicit RK
ERK vs. SDIRK

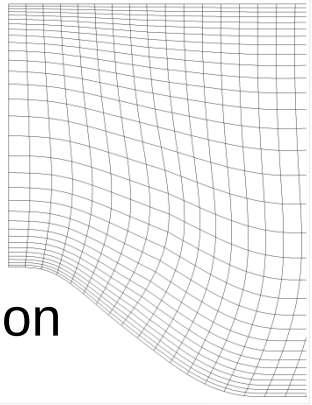
[Single coefficient diagonally implicit Runge-Kutta]



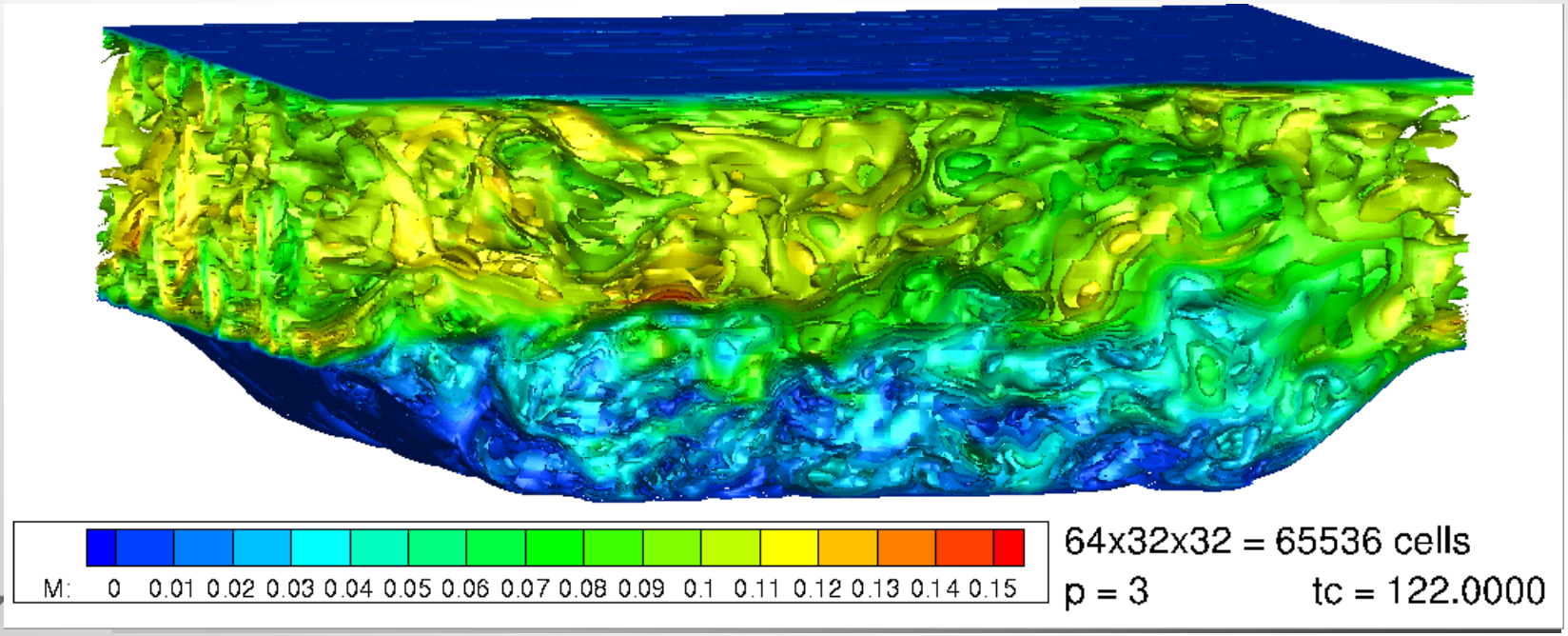
Scheme	t. step size	Run time	Note
ERK-4	0.001	**1.0**	reference
SDIRK-4	0.1	20.1	baseline
		1.3	freeze Jacobian for 1 time step, tol = 10^{-12}
		0.5	freeze Jac. for 1 time step, tol = 10^{-4}
		0.4	freeze Jac. for 2 time steps, tol = 10^{-4} („optimized“)
ESDIRK-3	0.05	0.8	„optimized“
ESDIRK-2	0.025	1.3	„optimized“

ILES for the 2D Periodic Hill test case at Re_b=2800

Quadratic grid
64x32x32 elem.
Global p-adaptation
ERK-4

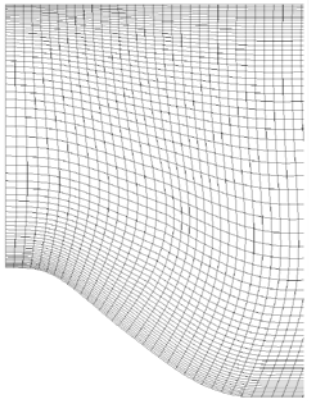


p	DoFs/eqn	tc	dtc	#steps
0	66 K	0 - 2	2e-4	10 K
1	262 K	2 - 32	1e-4	300 K
2	656 K	32 - 62	5e-5	600 K
3	**1.3 M**	62 - **122**	3.33e-5	1.8 M



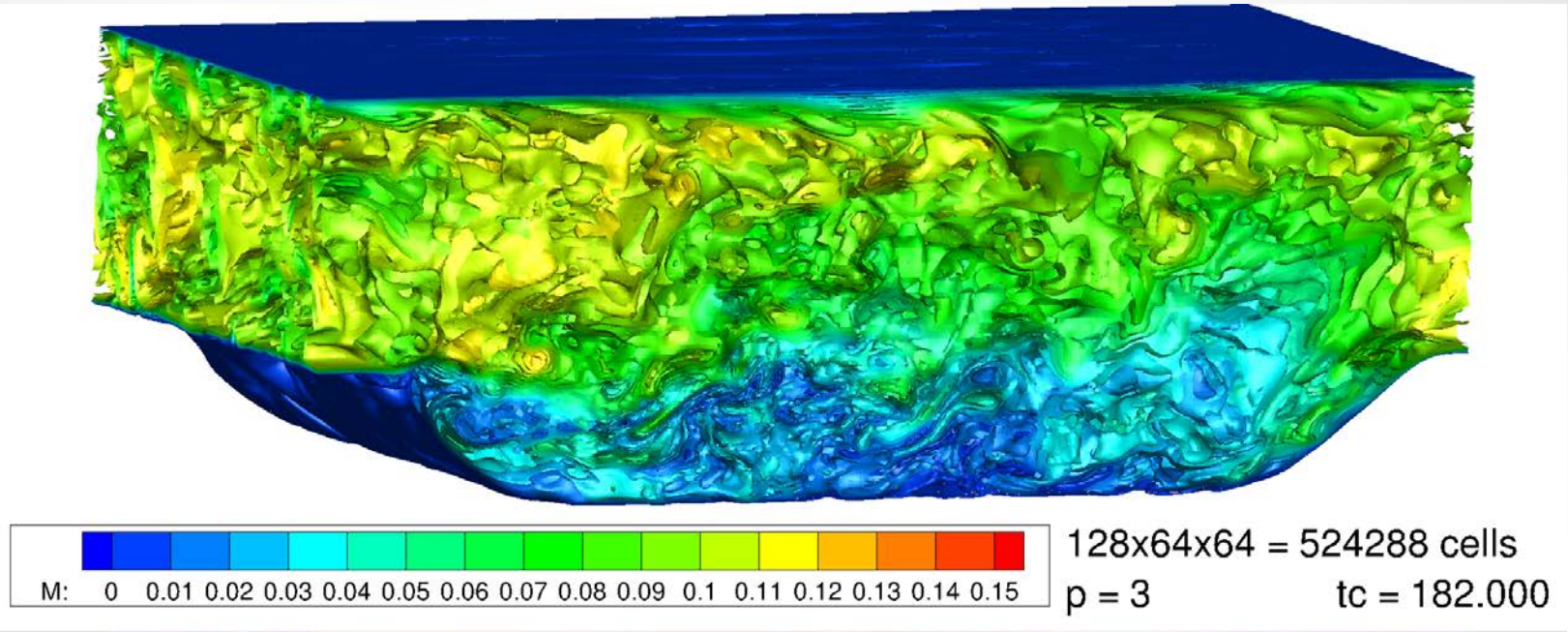
ILES for the 2D Periodic Hill test case at Re_b=2800

After global h-refinement:



Quadratic mesh
128x64x64 elem.
SDIRK-4

p	DoFs/eqn	tc	dtc	#steps
3	10.5 M	122 - **182**	1.66e-3	36000



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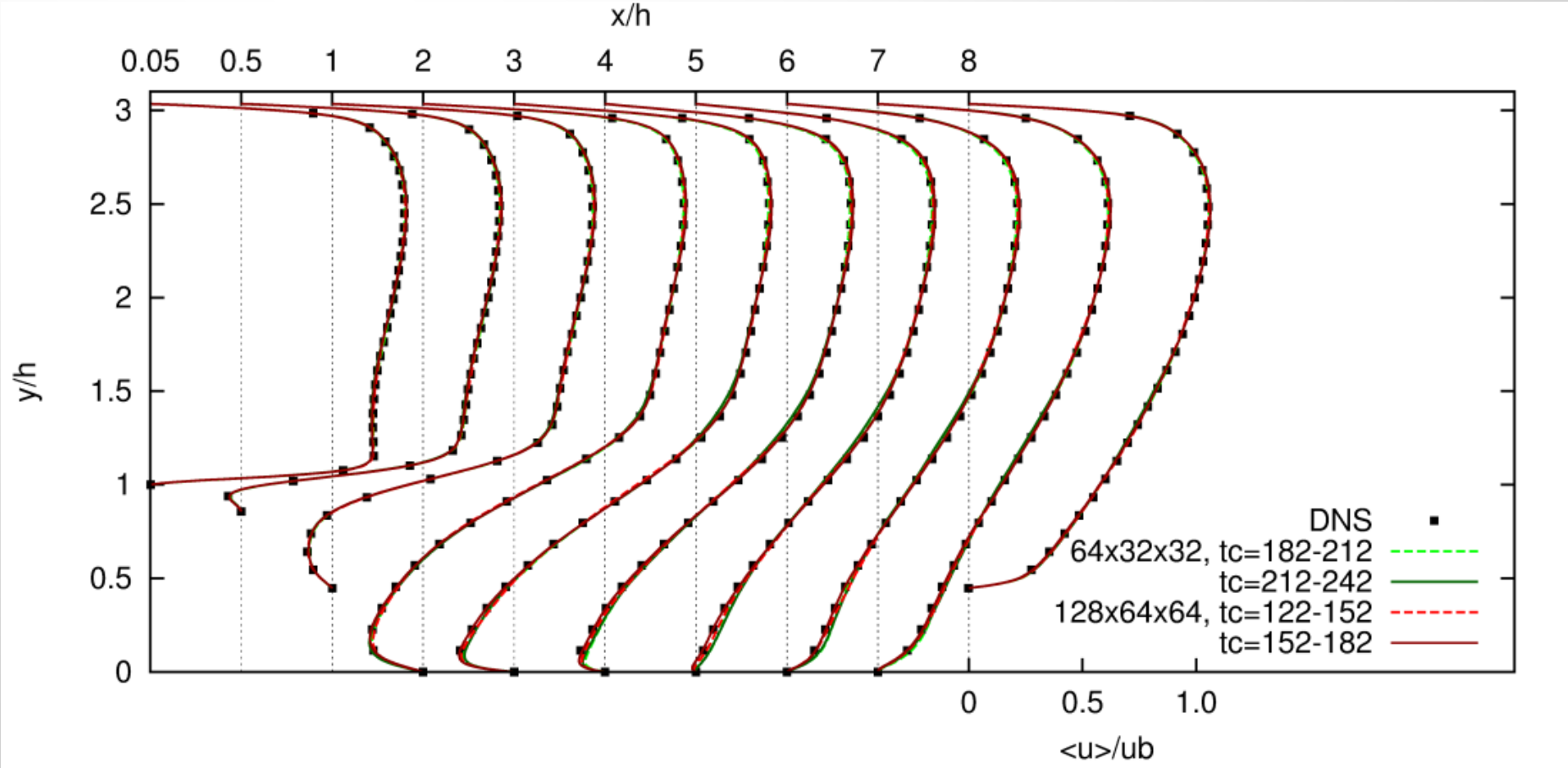

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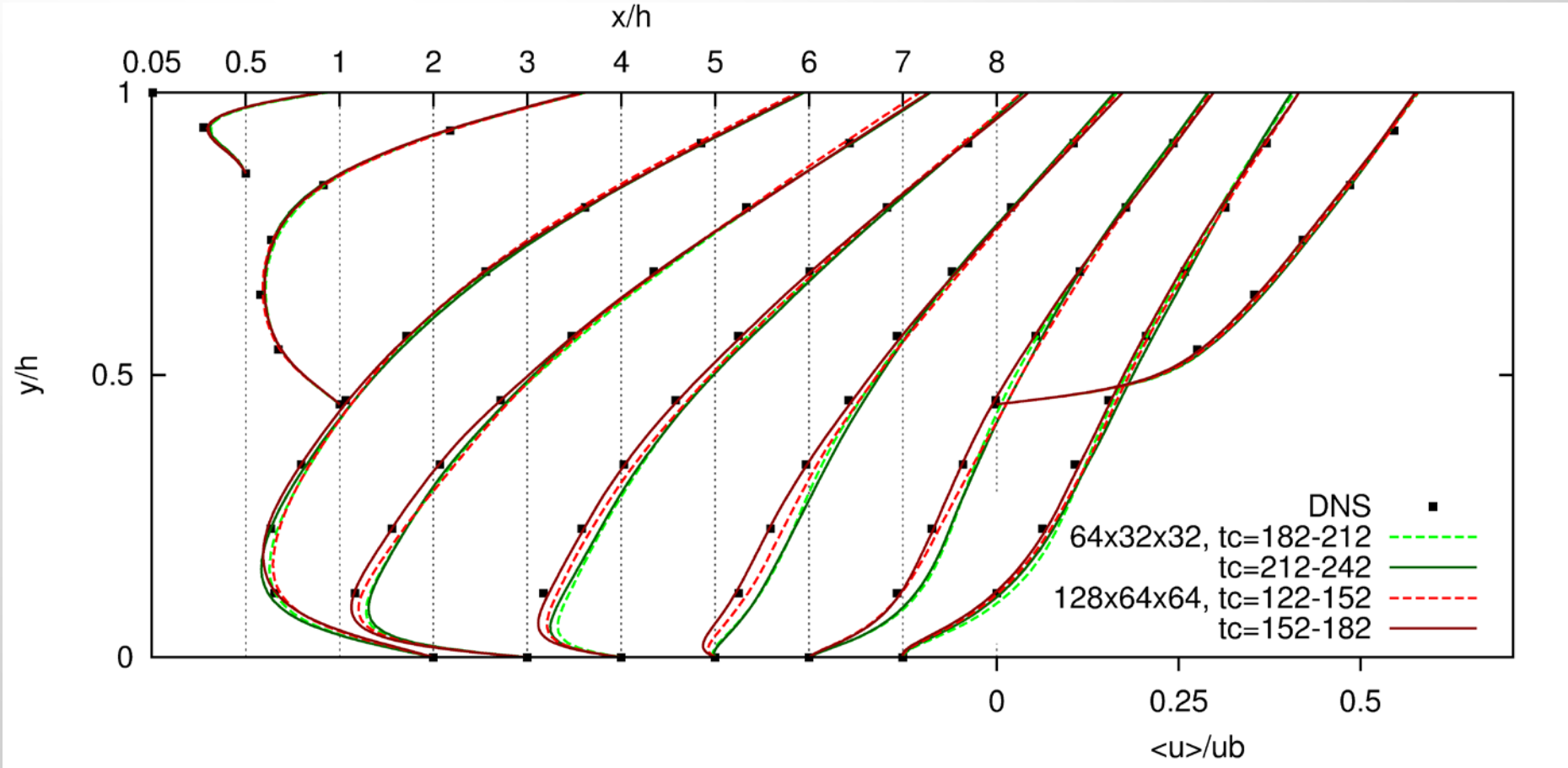
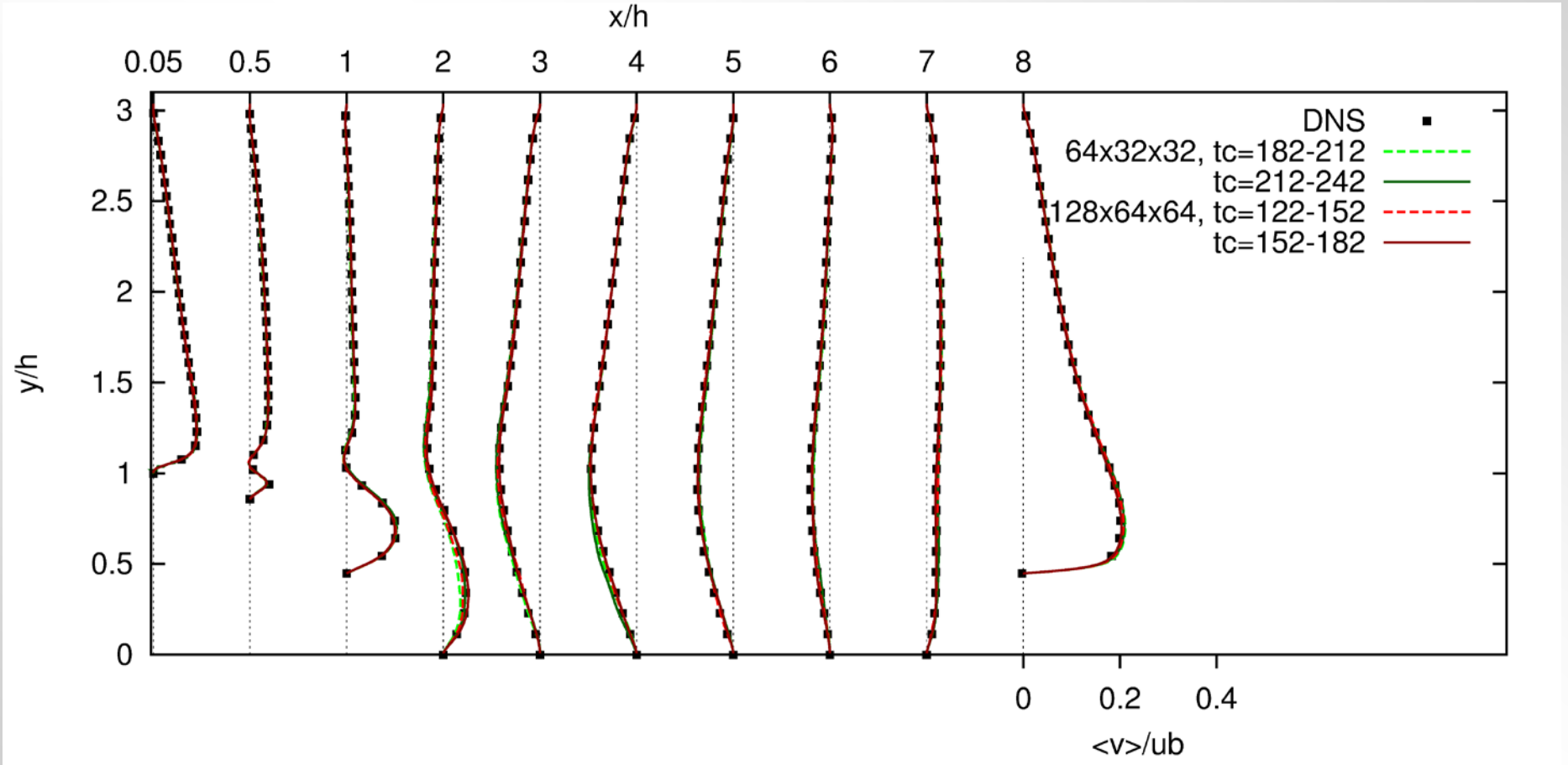


ILES for the 2D Periodic Hill test case at Re_b=2800



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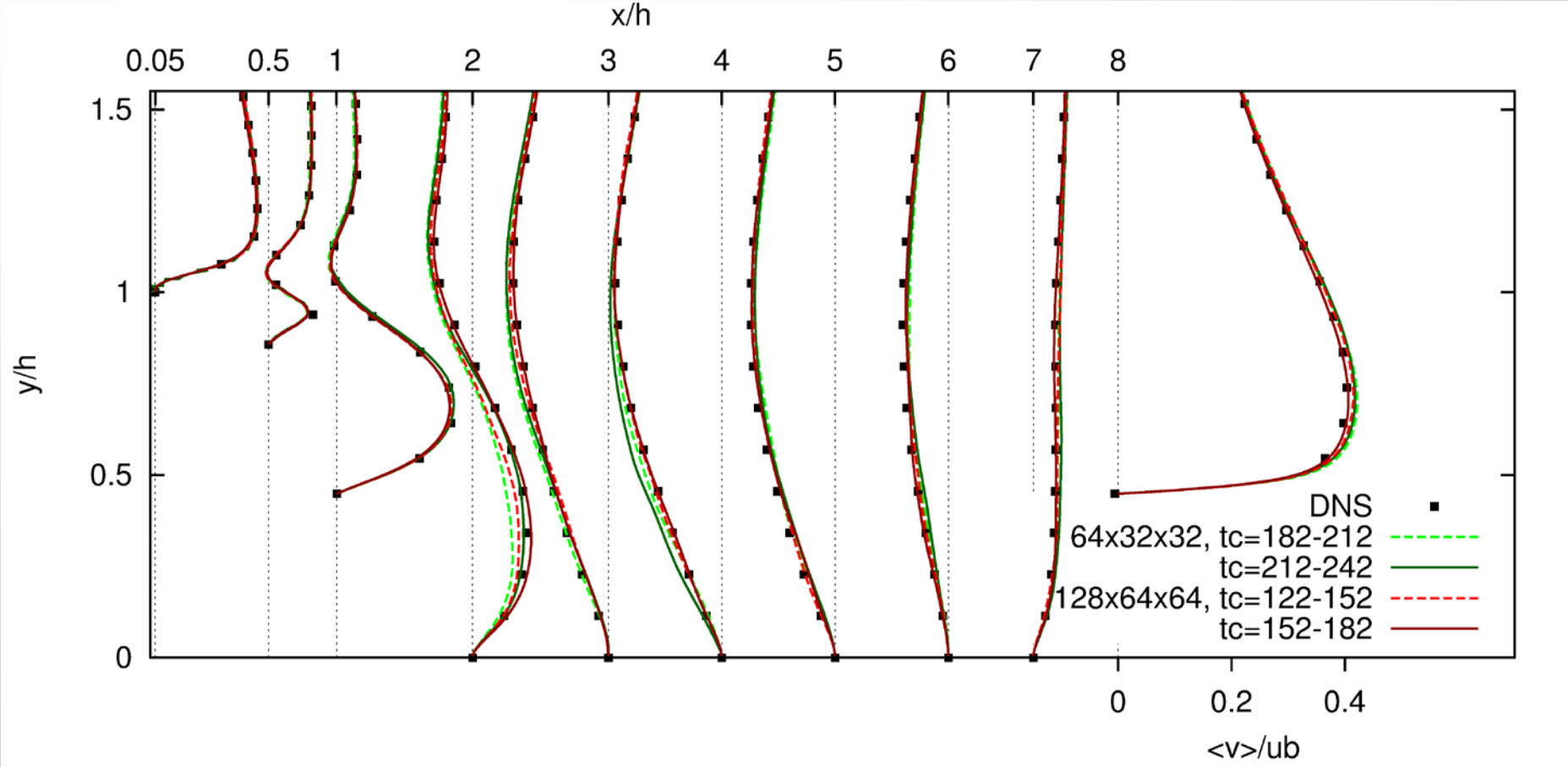


Channel flow, Re_δ = 6875, Re_τ(DNS)=392.24: Wall-resolved ILES

Prescribed data:

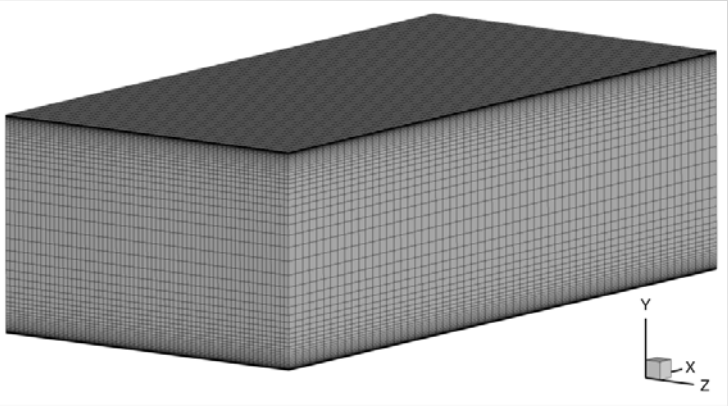
- Bulk Reynolds number $Re_\delta = U_{bulk} \cdot \delta / \nu$ and M=0.1

Measure quality of the (time-averaged) solution

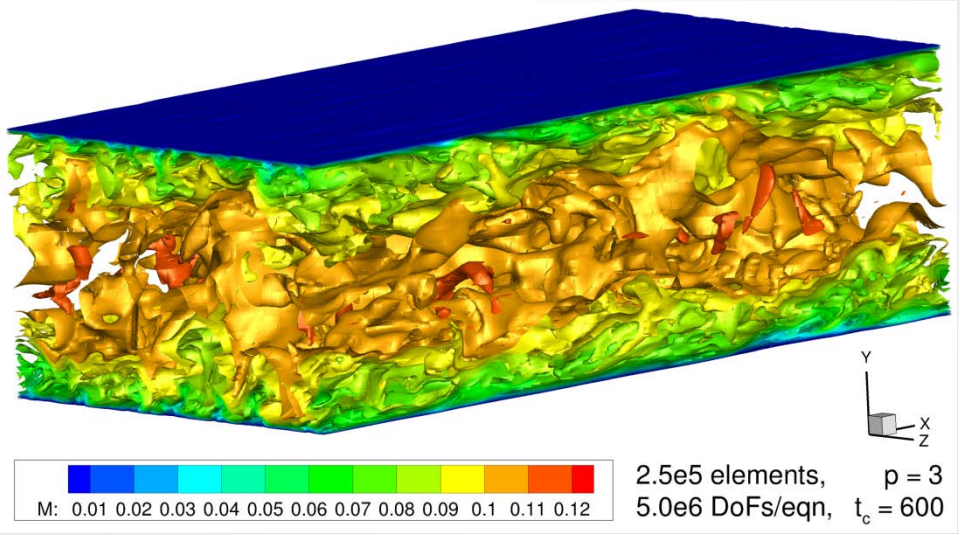
- by comparing against DNS data:
  - friction Reynolds number Re_τ
  - near-wall velocity profile $u^+(y^+)$

Computational mesh (gen. for hybrid RANS/LES [*]):

- 61 x 64 x 64 = 249856 elements
- Δx^+=41.15, Δy^+=0.78, Δz^+=19.61



p	DoFs/eqn	T_c	Re_τ
1	1.0 M	0 - 420 420 - 450	 297.65
2	2.5 M	450 - 510 510 - 540	 380.20
3	5.0 M	540 - 570 570 - 600 600 - 630	 392.30 391.41



[*] Probst, Löwe, Reuß, Knopp, Kessler. AIAA Journal, vol. 54, issue 10, pp. 2972-2987, 2016.

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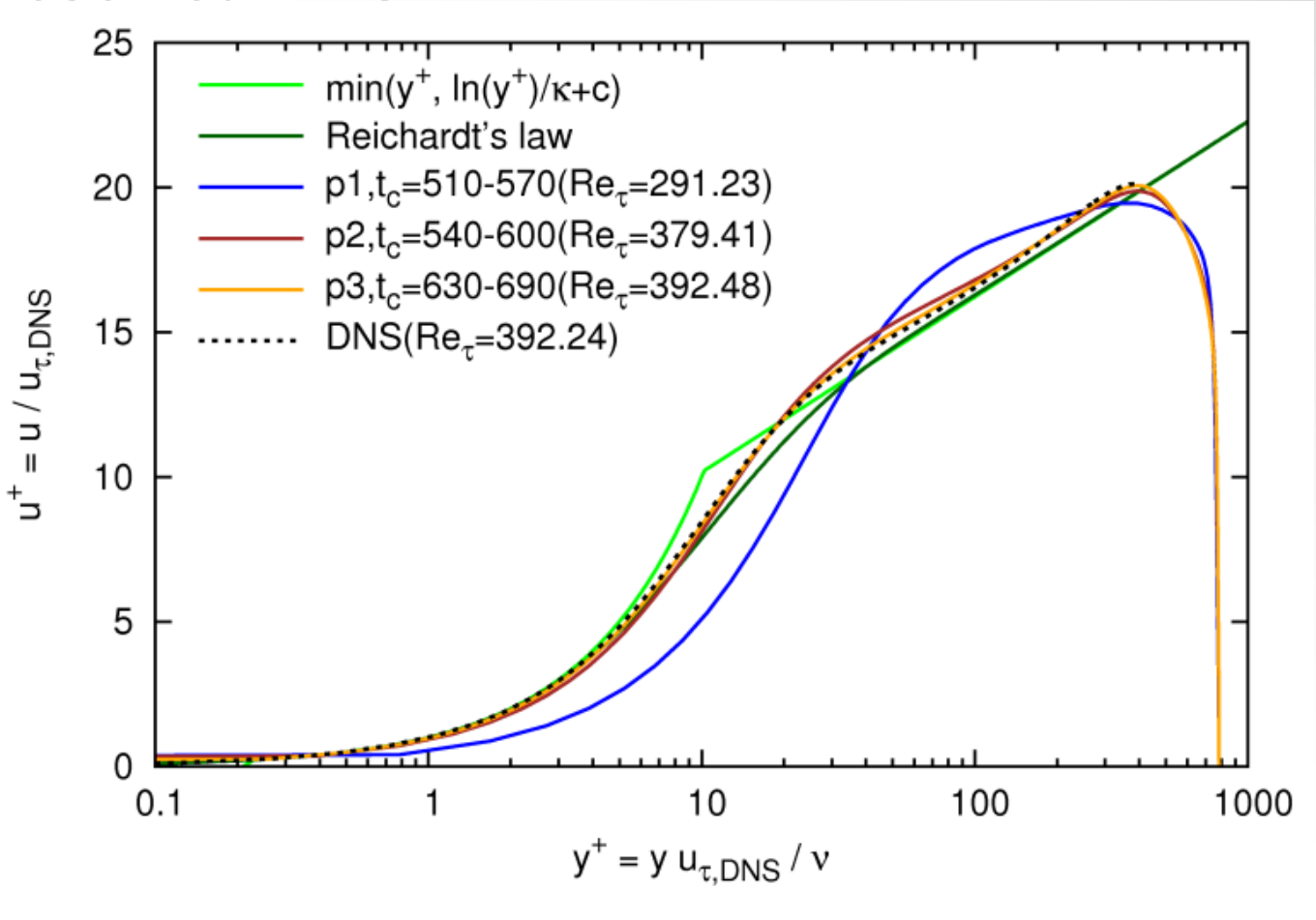

Re_δ = 6875, DNS[Moser, Kim, Mansour, 1999]: Re_τ = 392.24

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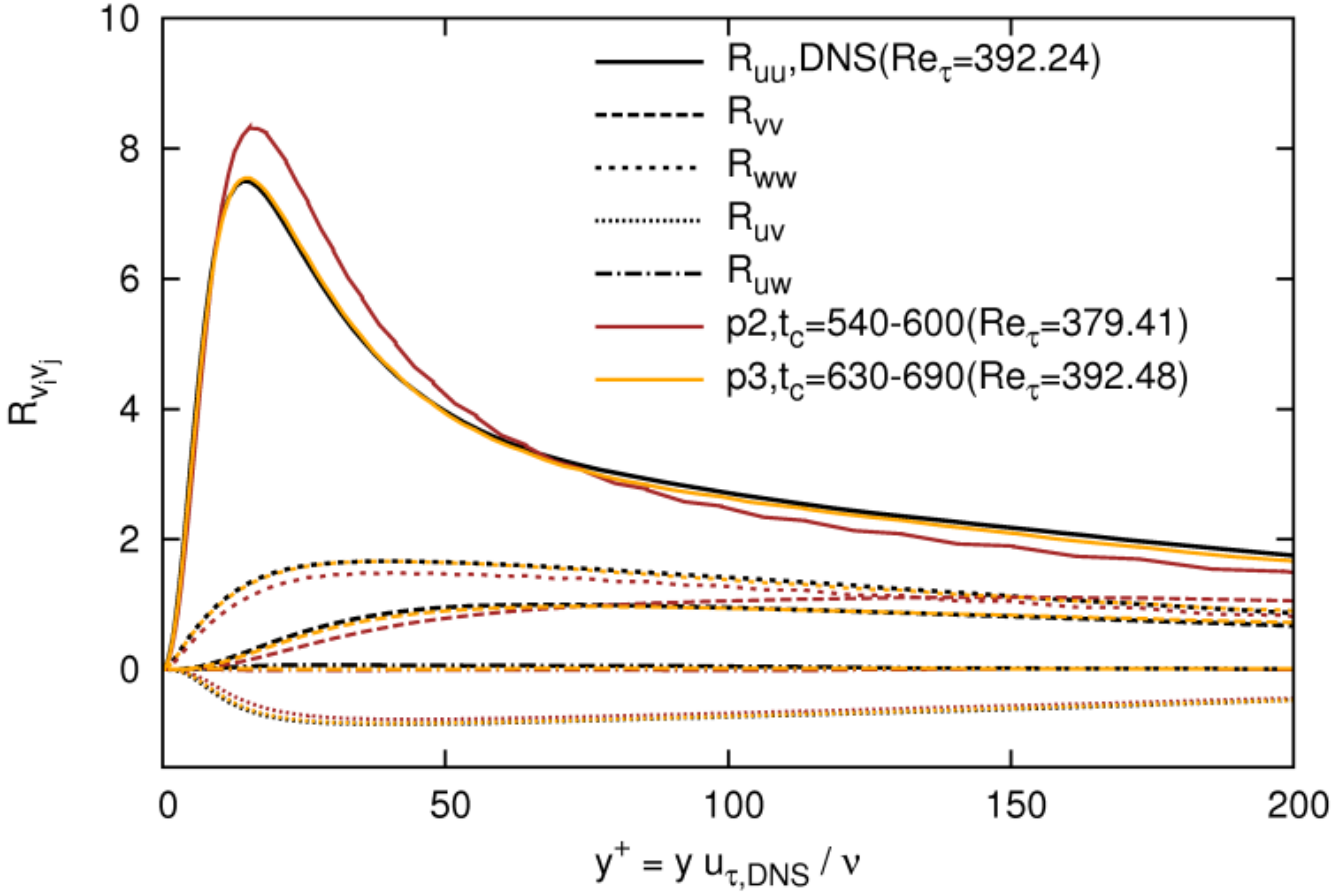

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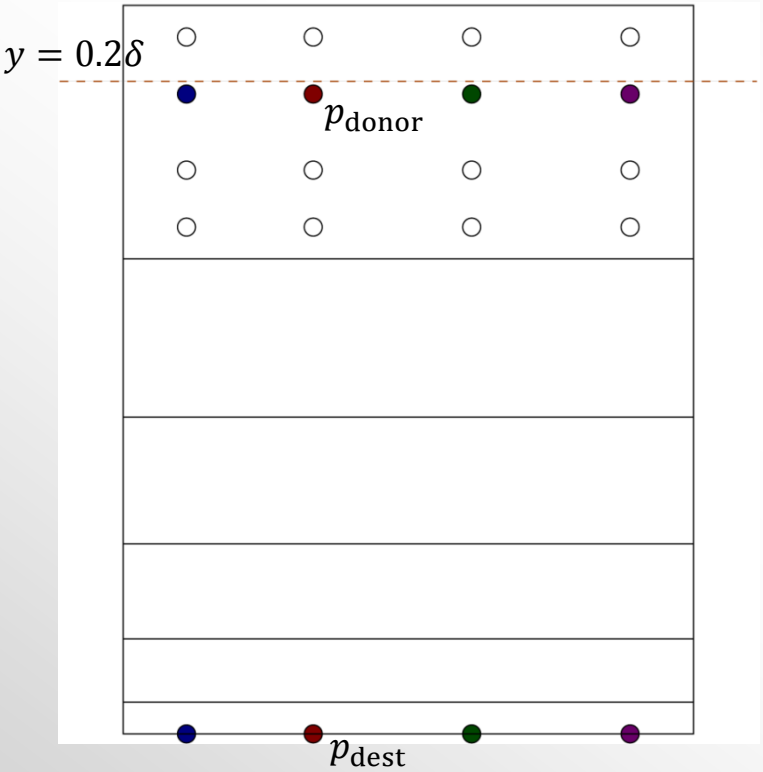
A wall-stress-model approach



For each boundary face integration point p_{dest}:

- Find a point p_{donor} normal to the wall in a distance of (approx.) y=0.2δ.
- From the solution (instantaneous flow field) at point p_{donor} take
  - tangential velocity $\mathbf{v}_t = (I - \mathbf{n} \otimes \mathbf{n})\,\mathbf{v}$
  - density ρ, kinematic viscosity ν, and
  - distance y = dist(p_{donor} , p_{dest}).
- Solve $u^+(y^+)=|\mathbf{v}_t|/u_\tau$ with $y^+=yu_\tau/\nu$, i.e.
  $F(u_\tau) = |\mathbf{v}_t|/u_\tau - u^+(y\,u_\tau/\nu) = 0$, for u_τ.
- Compute wall shear stress $\tau_w=\rho u_\tau^2$.
- In p_{face} apply slip-wall bc ($\mathbf{v}\cdot\mathbf{n}$=0) and a viscous numerical flux $\mathbf{n}\cdot F^v$ with prescribed τ_w:

Given the normal viscous flux

$$\mathbf{n}\cdot F^v(\mathbf{u},\nabla\mathbf{u}) = (\,0,\,(\tau\mathbf{n})_i\,,\,\mathbf{n}\cdot(\tau\mathbf{v}) + K\,\mathbf{n}\cdot\nabla T)$$

we split $(\tau\mathbf{n})_i = \tau_{ij}\,n_j$ into a wall normal and wall tangential part as follows

$$(\tau\mathbf{n}) = (\tau\mathbf{n})_n + (\tau\mathbf{n})_t \quad \text{with } (\tau\mathbf{n})_t = (\,I - \mathbf{n}\otimes\mathbf{n})\,(\tau\mathbf{n}),$$

replace $(\tau\mathbf{n})_t$ by $(\tau\mathbf{n})_t^{\text{wm}} = -\tau_w\hat{\mathbf{v}}_t$ and use the adiabatic condition $K\,\mathbf{n}\cdot\nabla T$=0.

Near-wall velocity profiles

u(y) scaled with the friction velocity u_τ and the kinematic viscosity ν:

$u^+(y^+)$ with $u^+ = u/u_\tau$ and $y^+ = y\, u_\tau/\nu$

Algebraic near-wall velocity profiles:

- Logarithmic law-of-wall (log-law): $u^+(y^+) = \min(\, y^+, \ln(y^+)/\kappa + c\,)$
  - Von Karman constant $\kappa \in [0.38, 0.41]$ and $c \in [4.1, 5.1]$
  - Take $\kappa = 0.38$ and $c = 4.1$ [Österlund et al., 2000]

- Reichardt's law-of-wall: $u^+(y^+) = \ln(1+\kappa y^+)/\kappa + A(1- e^{-y+/B} - y^+/B\, e^{-y+/C})$
  - $A \in [6.6, 7.8]$, $B = 11$, $C = 3$
  - Take $A = c - \ln(\kappa)/\kappa$ [Frere, de Wiart, Hillewart et al., 2017]

- Spalding's (inverse) law-of-wall: $y^+(u^+)$

„Exact" near-wall velocity profile:

- Moser, Kim, Mansour(1999): DNS of turbulent channel flow up to $Re_\tau = 590$.

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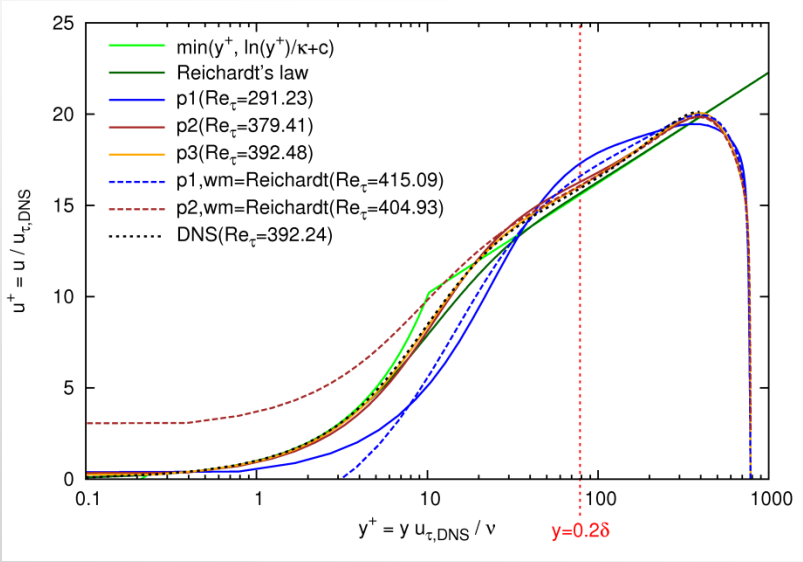
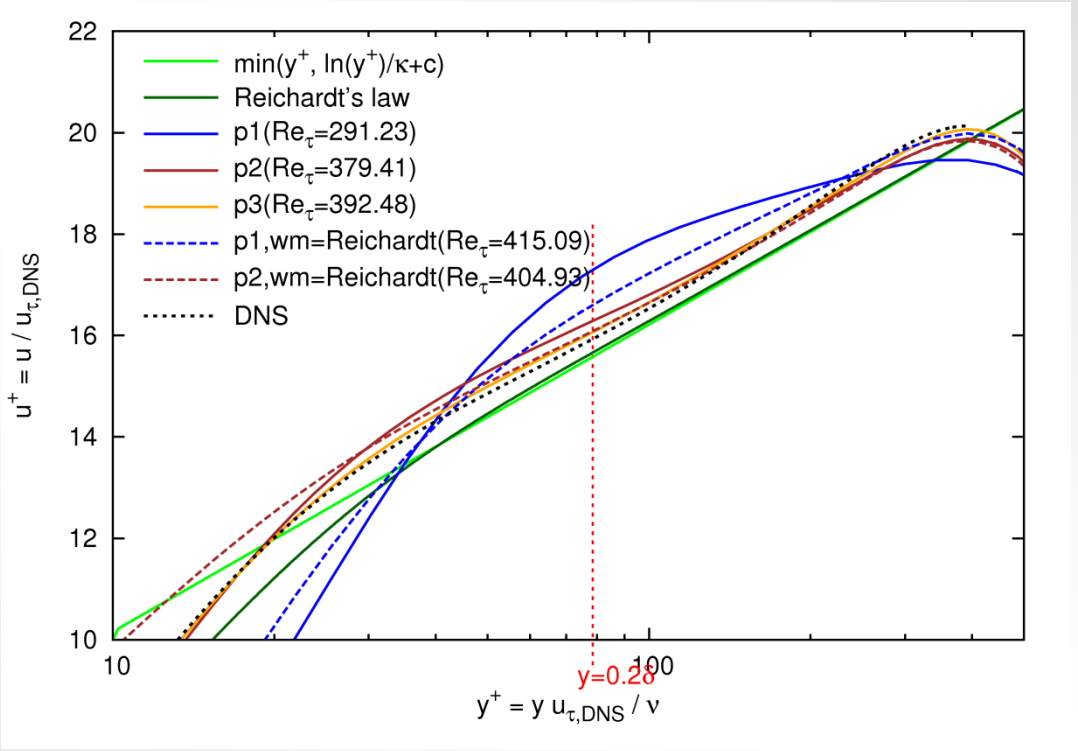
Channel flow, Re_δ = 6875, Re_τ(DNS)=392.24: Wall-modelled ILES vs. wall-resolved ILES

p	DoFs/ eqn	ILES Re_τ	WM-ILES $u^+(y^+)$=Reichardt Re_τ	WM-ILES $u^+(y^+)$=DNS data Re_τ	DNS [Moser et al.] $Re_{\tau,DNS}$
1	1.0e6	293.08 (-25.3%)	415.24 (5.9%)	408.98 (4.3%)	
		291.23 (-25.8%)	418.81 (6.8%)	409.89 (4.5%)	
			415.09 (5.8%)	408.47 (4.1%)	
2	2.5e6	379.88 (-3.2%)	403.75 (2.9%)	400.42 (2.1%)	392.24
		379.41 (-3.3%)	404.93 (3.2%)	399.78 (1.9%)	
3	5.0e6	391.86 (-0.1%)			
		392.48 (0.06%)			

Table 4: Channel flow at $Re_\delta = 6875$: Comparison of friction Reynolds numbers for ILES and two versions of WM-ILES (once $u^+(y^+)$ is given by Reichardt's law and once given by DNS data) compared to $Re_{\tau,DNS}$ [Moser et al., 1999]. Re_τ values are computed from solutions averaged over 60 CTU (for ILES) and 120 CTU (for WM-LES). Deviations of computed Re_τ values from $Re_{\tau,DNS}$ are given as percentages.

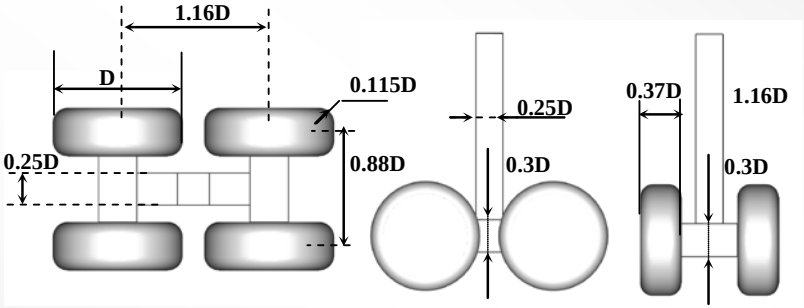
Channel flow, Re_δ = 6875, Re_τ(DNS)=392.24: Wall-modelled ILES vs. wall-resolved ILES





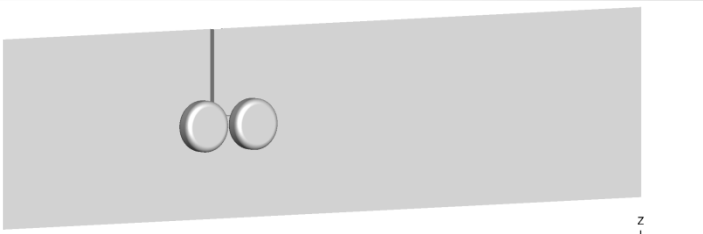
Boeing Rudimentary landing gear

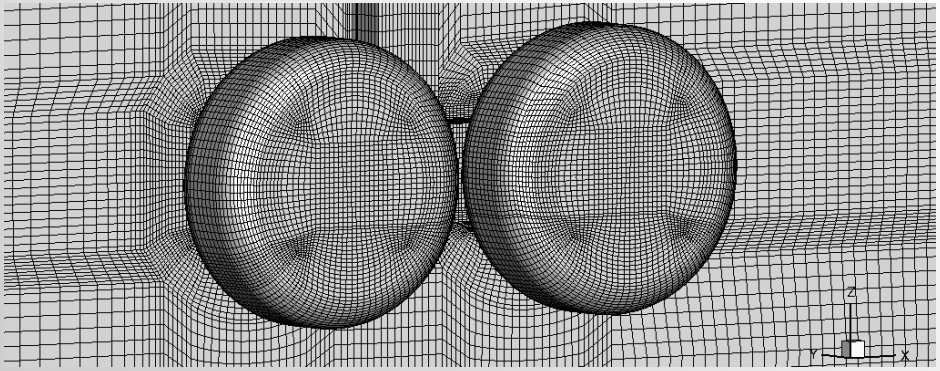
U=40m/s, (M≈0.12), Re=UD/v≈10^6

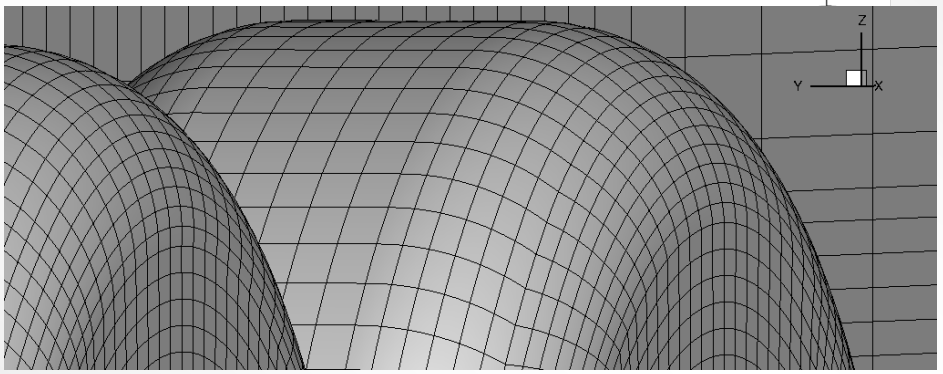


Grids derived from structured grid (ATAAC)

- coarse(115k), medium(924k), fine(7.4M)







medium quadratic grid with 924k elements

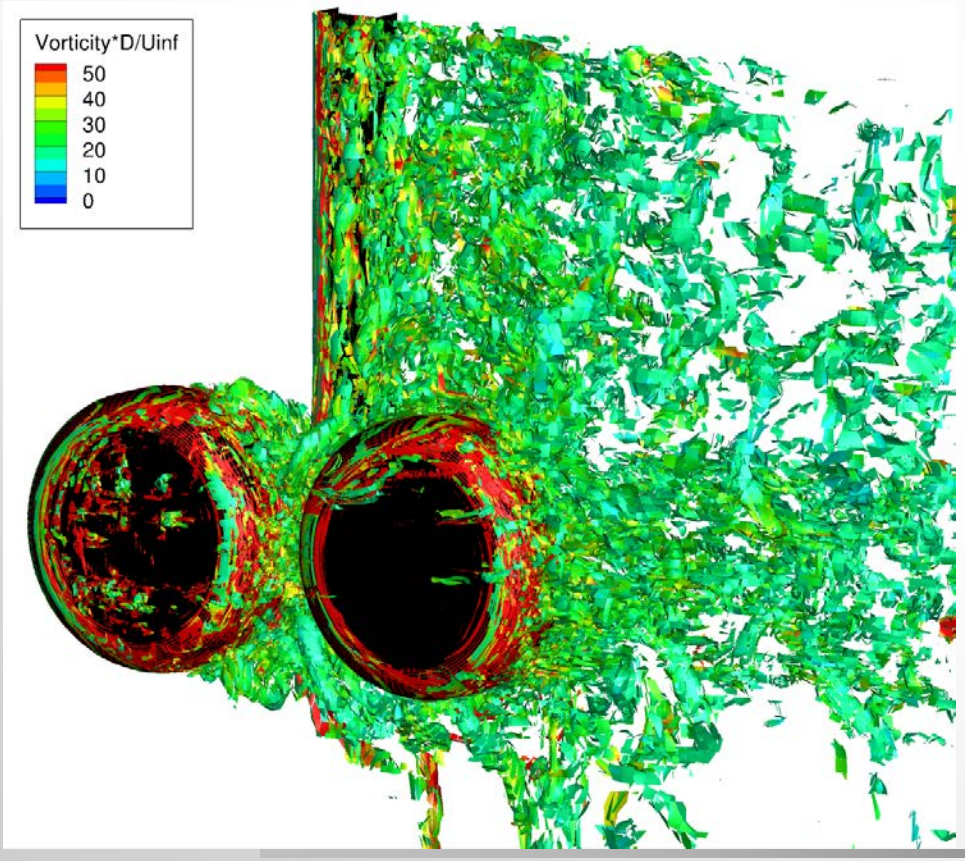
ILES for the Boeing RLG

For p=2 on medium grid:

- ERK-4: 50e6 steps/CTU
- SDIRK-4: 200 steps/CTU

Computational time(ERK/SDIRK)=100*

grid	# el	p	DoFs/eqn	Δt_c	t_c
coarse	115 K	1	0.5 M	0.01	0-30
medium	924 K	1	3.7 M	0.01	30-60
		2	**9.2 M**	**0.005**	**60-66.6**



Medium quadratic grid
p=2 (3rd order DG)
local Lax-Friedrichs flux

Instantaneous flow field at tc=65.6:

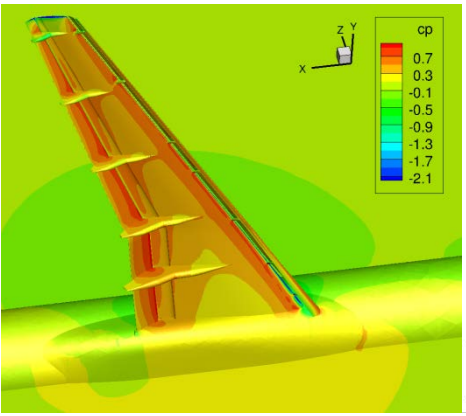
- Iso-surfaces of Q criterion
  $Q\,(D/U_\infty)^2=50$
  colored with the vorticity magn.

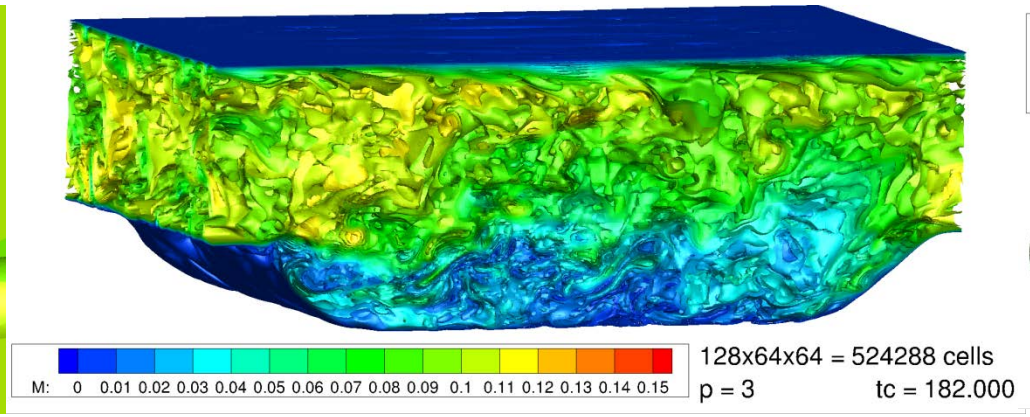
Computation broke after tc=66.6
Next steps: Use

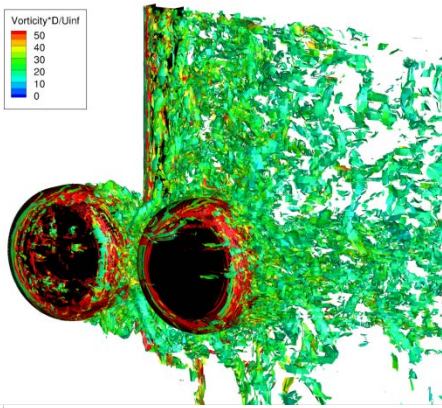
- BCs based on Riemann invariants
- Sponge layer near outflow?
- WM-ILES

Summary

- DG discretization of equations, integral quantities and local quantities
- Details on discretization settings and results for RANS and ILES:
  - RANS and Wilcox-kω for the DLR-F11 high lift configuration
  - Computational time(ERK/SDIRK)=2.5(TGV), 100*(Boeing RLG)
  - **Wall-resolved ILES** for 2D periodic hill and channel flow with results very close to DNS data
  - **Wall-modelled ILES** for channel flow with significant improvement over ILES
- Current state of ILES computations for the Boeing rudimentary landing gear







RANS-kω with p=2

ILES with p=3

ILES with p=2